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Rules for localized overlappings and intersections of p-branes

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Abstract: We determine an intersection rule for extremal p-branes which are localized in their relative transverse coordinates by solving, in a purely bosonic context, the equations of motion of gravity coupled to a dilaton and n-form field strengths. The unique algebraic rule we obtained does not lead to new solutions while it manages to collect, in a systematic way, most of the solutions (all those compatible with our ansatz) that have appeared in the literature. We then consider bound states of zero binding energy where a third brane is accomodated in the common and overall transverse directions. They are given in terms of non-harmonic functions. A different algebraic rule emerges for these last intersections, being identical to the intersection rule for p-branes which only depend on the overall transverse coordinates. We clarify the origin of this coincidence. The whole set of solutions in ten and eleven dimensional theories is connected by dualities and dimensional reductions. They are related to brane configurations recently used to study non-perturbative phenomena in supersymmetric gauge theories.

Keywords: Branes in String Theory, Supergravity Models.

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Contents

1. Introduction

In the last two years, there has been a growing amount of evidence in favour of the possible unification of the five known superstring theories due to the existence of non-perturbative duality symmetries [1, 2]. An eleven dimensional theory has been conjectured, M-theory, that enables us to understand the duality properties of string theory in a unified way [3] and whose low-energy limit is given by $D = 11$ supergravity. A microscopic description of this theory has been proposed in terms of the large N limit of a supersymmetric quantum mechanical system of $N \times N$ matrices [4]. It has been widely stressed in the literature that one of the key ingredients that leads to the identification of duality symmetries is given by a proper knowledge of the solitonic spectrum of these theories, this involving p-dimensional excitations called p-branes.

It is known that $D = 11$ supergravity contains M2-branes and M5-branes, both playing a major rôle in the dynamics of M-theory. These branes preserve one half of the supersymmetries, hence they are BPS states. In string theory, an entire zoo of BPS p-brane solutions has been studied in the last few years. In type II theories, there are two kinds of p-branes depending on the sector of the world-sheet theory where the charge that they carry is originated. The NS-NS sector contains the fundamental string and the solitonic fivebrane, the NS5-brane. The p-branes that carry R-R charge have been shown to be described by hypersurfaces where open strings can end, called D-branes [5].

One of the most interesting aspects of this variety of branes is given by the possibility of constructing composite brane configurations –starting from the previously mentioned basic bricks– that preserve a certain amount of supersymmetries. This kind of configurations has led very recently to a large number of celebrated results both in supergravity and supersymmetric gauge theories. Intersections of a large number of D-branes has made possible to identify and count the microscopic states corresponding –after compactification– to certain black hole geometries, in complete agreement with the semiclassical entropy [6, 7, 8]. Another remarkable fact is that some of the non-perturbative properties of supersymmetric gauge theories in various spacetime dimensions were found to have a natural explanation in string theory, by studying the low-energy dynamics of a certain class of intersecting brane configurations [9, 10, 11, 12, 13, 14]. In this scenario, it is of great interest to derive, on general grounds, a set of rules that states and classifies all possible brane configurations that preserve a specific amount of supersymmetries.

For the case of D-branes, by using the string theory representation of the branes and duality, certain rules were derived in Refs. [15, 16, 17]. The case of intersecting M-branes was considered in [18] and the study of intersecting p-branes starting from eleven dimensional supergravity has led to the formulation of the so-called harmonic superposition rule [19]. Intersection of both M-branes and D-branes were subsequently classified in Ref. [20]. Another derivation of the intersection rules, not based on supersymmetry arguments, was done by requiring that p-brane probes in q-brane backgrounds feel no force and can thus create bound states with vanishing binding energy [21].

More recently, a general rule determining how extremal branes can intersect in a configuration with zero binding energy has been derived in Ref. [22].[1] This rule is obtained from the bosonic equations of motion of the low-energy theory and unifies in a remarkably simple way all classes of branes in any spacetime dimension.[2] The kind of configurations considered there are translational invariant in all directions tangent to any participating brane. This is an appropriate restriction if one is to consider toroidal compactification in which each p-brane is wrapped on a p-cycle to end with a solution representing an extreme black hole of the compactified supergravity theory. In spite of the fact that it is useful for a wide range of intersecting brane configurations, several of the most interesting cases that recently appeared in the literature in the context of supersymmetric gauge theories are excluded, namely, the localized overlappings (or intersections) of p-branes.

[1] Also, following a slightly different approach, in Ref. [23].

[2] It was also generalized in Ref. [24] to include intersections of non-extreme p-branes.

Two overlapping branes can be obtained from intersecting branes by separating each brane in a direction transverse to the remaining one. This sort of configurations are included in the analysis of Ref. [22]. However, as pointed out in Refs. [18, 25, 26, 27], this kind of solutions are not true overlappings or intersections in the sense that the harmonic function corresponding to a given brane is translational invariant in the directions tangent to the other brane or, what is the same, each brane is delocalized in its relative transverse space. Instead, we would like to consider in this paper p-brane configurations with localized intersections which are, as we briefly argued above, relevant for the study of strong coupling phenomena in supersymmetric gauge theories. To our knowledge, the first example of this kind of configuration was first constructed by Khuri [28] while studying a string-like soliton solution in heterotic string theory, though the interpretation as a system of localized intersecting branes was not discussed there. Recently, Gauntlett, Kastor and Traschen [25] have clarified this issue showing that it corresponds to two NS5-branes intersecting on a string in type II string theory. As also shown in Ref. [25], when uplifting this solution to eleven dimensions, one is faced with a configuration of two M5-branes overlapping on a string that has a striking characteristic: although the harmonic functions do depend on the relative transverse directions, they are translational invariant in the remaining overall transverse direction. There may be solutions in which the M5-branes are also localized in the overall transverse direction but, if so, they shall not be given by the harmonic superposition rule [29].

It is clear that, starting from this eleven dimensional configuration, a large class of solutions would be accesible, whose distinguishing characteristic will be that all branes, while being localized in the relative transverse space, are delocalized in the overall transverse directions. This defines an appropriate kind of configurations in the context of the recently developed brane techniques for the study of non-perturbative phenomena and dualities of supersymmetric gauge theories [9, 10, 11, 12, 13, 14]. In fact, these techniques involve arrangements of flat p-branes with *localized* intersections, such that the field theory on the world-volume of the branes has the desired gauge symmetry, matter content and degree of supersymmetry in the spacetime dimension which is specifically chosen (three in [9], four in the rest of the papers cited above). In these approaches, one of the constituent brane is finite in a given direction in which it is stretched between other much heavier branes. To obtain explicit solutions of supergravity displaying this behaviour is, of course, an involved task. However, a first approach in this direction is given by finding generic configurations of infinite p-branes with localized intersections, that preserve a certain number of supersymmetries. Indeed, as discussed in Ref. [27], under certain circumstances it is possible to think of these configurations as corresponding to a p-brane stretched between other branes.

In this paper we establish some rules for this kind of intersections. We find these rules purely from the bosonic equation of motion of the low-energy theory, in the spirit of Ref. [22]. The rule corresponding to localized intersections of p-branes is obtained in section 2 by solving the equations of motion using an ansatz that accounts for the properties of extremality and zero binding energy. The whole set of classical fields is written in terms of a number of functions equal to the number of single branes involved in the configuration. It also implies the preservation of a specific amount of supersymmetry. In section 3, we analize the resulting solutions in ten and eleven dimensional theories and comment on their relation with certain brane configurations which are relevant for the study of non-perturbative supersymmetric gauge theories. It is important to point out that *there are no new solutions within our ansatz*. Our rule manages to collect many of the solutions that have appeared in the literature[3] in a unique algebraic expression which is fully derived within the bosonic sector of the theory. These solutions are connected among themselves by a chain of dualities and dimensional reductions, a fact which is at the root of the possibility to build a unique expression that accounts for an entire family of solutions. We think that most of the interest of our approach is given by the fact that it provides a systematic alternative procedure –with respect to the usual one relying on Γ-matrices algebra (see, for example, [29] and references therein)– to build and classify intersecting brane configurations. Indeed, generalizations of our ansatz should lead to the appearance of new solutions as, *e.g.*, non-extremal branes, branes at angles, (p,q) webs, etc.

In section 4, we show that a third brane can be added into the configuration with vanishing binding energy. We derive the corresponding intersection rule. We impose some new conditions on the metric that reproduce the extremality nature of the configuration. In section 5, we analyze the solutions that emerge from these intersection rules, which again fit several known cases in the literature, and comment on their close relation with some of the configurations used in the brane approach to strong coupling phenomena of supersymmetric gauge theories. These configurations, as well as other pairwise intersections that can be obtained from them, are given in terms of a non-harmonic function. We show that they provide a generalization of the class of solutions discussed in Ref. [22], thus obeying the same intersection rule. Finally, in section 6, we discuss our results and make some futher comments. In the Appendices, the detailed expression for the Ricci tensor is given both for the two brane and three brane cases, and the way the extremality condition appears in our framework is clarified.

2. Localized intersection of two branes

Consider the following general expression for the bosonic sector of the low-energy effective action corresponding to superstring theory in any spacetime dimension D, $D \leq 11$,

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \sum_{A=1}^{Q} \frac{1}{2n_A!} e^{a_A \phi} F_{n_A}^2 \right) , \tag{2.1}$$

[3]We should mention that there are also solutions representing localized intersections of p-branes that could not be reached from our starting ansatz (*e.g.* those obtained in Ref. [30] from hyper–Kähler manifolds).

The action includes gravity, a dilaton and Q field strengths of arbitrary form degree $n_A \leq D/2$ and coupling to the dilaton a_A. The metric is expressed in the Einstein frame. There may be Chern-Simons terms in the action but we omit them as they are irrelevant for the kind of solutions we will concentrate on. Although we take the spacetime to have a generic dimension D, this action is most suitable for describing the bosonic part of $D = 10$ or $D = 11$ supergravities. The equations of motion can be written in the following form:

$$R^\mu{}_\nu = \frac{1}{2}\partial^\mu\phi\partial_\nu\phi + \sum_{A=1}^{Q} \Theta_A{}^\mu{}_\nu\,, \tag{2.2}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu)\phi = \sum_{A=1}^{Q} \frac{a_A}{2n_A!} e^{a_A\phi} F^2_{n_A}\,, \tag{2.3}$$

$$\partial_{\mu_1}\left(\sqrt{-g}e^{a_A\phi}F^{\mu_1\ldots\mu_{n_A}}\right) = 0\,, \tag{2.4}$$

where $\Theta_A{}^\mu{}_\nu$ is the contribution to the stress-energy tensor corresponding to the n_A-form,

$$\Theta_A{}^\mu{}_\nu = \frac{1}{2n_A!}e^{a_A\phi}\left(n_A F^{\mu\rho_2\ldots\rho_{n_A}}F_{\nu\rho_2\ldots\rho_{n_A}} - \frac{n_A-1}{D-2}F^2_{n_A}\delta^\mu{}_\nu\right)\,. \tag{2.5}$$

We must supplement the equations of motion by imposing the Bianchi identities to the n_A-forms,

$$\partial_{[\mu_1}F_{\mu_2\ldots\mu_{n_A+1}]} = 0\,. \tag{2.6}$$

as they are field strengths of $(n_A - 1)$-form potentials. We are interested in classical solutions describing a pair of p-branes that are translationally invariant in the overall transverse directions but are localized in the relative transverse coordinates. Thus, we set (for simplicity) all but two field strengths to zero (a condition that will be relaxed in section 4).

Let us now specialize to a particular form of the metric which is a slight generalization of the p-brane ansatz, and lead us to obtain the class of configurations we want to deal with. The line element is given by

$$\begin{aligned} ds^2 = &- B^2dt^2 + C^2\delta_{ij}ds^ids^j + X^2\delta_{ab}dx^adx^b + Y^2\delta_{\alpha\beta}dy^\alpha dy^\beta \\ &+ W^2\delta_{\mu\nu}dw^\mu dw^\nu\,, \end{aligned} \tag{2.7}$$

where the s_i's span the intersection, $i, j = 1\ldots\bar{q}$, the x_a's and y_α's are the relative transverse coordinates $a, b = 1\ldots p_1$ and $\alpha, \beta = 1\ldots p_2$, whereas the w_μ's are the overall transverse coordinates $\mu = 1\ldots p_t$. The functions B, C, X, Y and W depend only on the relative transverse coordinates x^a, y^α. It is clear that the relation $\bar{q} + p_1 + p_2 + p_t = D - 1$ must be satisfied. Furthermore, we will consider solutions that allow a factorization of the form:

$$\mathcal{F}(x^a, y^\alpha) \equiv \mathcal{F}_x(x^a)\mathcal{F}_y(y^\alpha)\,, \tag{2.8}$$

for the whole set of functions. These solutions will represent a q_1-brane and a q_2-brane with a $\bar{q}$-dimensional localized intersection, being $q_A = p_A + \bar{q}$.

For the n_A-form field strengths, we can generally make two kinds of ansätze. The electric ansatz is done asking that the Bianchi Identities are trivially satisfied. Consider, for example, an electrically charged q_1-brane. It couples naturally to a q_1+2-form, F_{n_1},

$$F_{0i_1\ldots i_{\bar{q}}a_1\ldots a_{p_1}\alpha} = \epsilon_{i_1\ldots i_{\bar{q}}}\epsilon_{a_1\ldots a_{p_1}}\partial_\alpha E_1\,. \tag{2.9}$$

For a magnetic brane, on the other hand, one asks that the equations of motion for the field strength (2.4) are trivially satisfied. Thus, a magnetically charged q_1-brane couples to a $(D-q_1-2)$-form that can be written as follows form,

$$F^{\mu_1\ldots\mu_{p_t}\alpha_1\ldots\alpha_{p_2-1}} = \frac{1}{\sqrt{-g}}e^{-a\phi}\epsilon^{\mu_1\ldots\mu_{p_t}}\epsilon^{\alpha_1\ldots\alpha_{p_2-1}\alpha}\partial_\alpha E_1\,. \tag{2.10}$$

We remark here on the fact that the derivative is always taken with respect to directions which are *perpendicular* to the respective brane. This is in accordance with the cases considered in [25] and with the ansätze of [22] for intersecting branes. Also the dilaton depends on the relative transverse coordinates.

Let us now discuss in some detail the next ansätze that we will make in order to solve the equations of motion (2.2)–(2.4). The Ricci tensor that corresponds to the metric (2.7) (which is computed in an Appendix) displays several terms that mix non-trivially the different components of the metric. On the other hand, we should be able to write B, C, X and Y in terms of a pair of functions in order to make the distinction between the constituent branes. These functions are supplemented by the metric component relative to the overall transverse space W, the dilaton ϕ and the set of functions E_A. We are thus forced to impose further constraints on our configuration. If viewed as a classical configuration of supergravity, the conditions that one should impose would be the vanishing of the supersymmetry transformations –corresponding to a given infinitesimal parameter η that satisfies certain chirality constraints– for all the fermions. This amounts to the preservation of some of the supersymmetries (those related to the particular parameter η). In our approach, we are not going to analyze the whole content of the supergravity theory that is behind (2.1). We should, instead, take profit of the signals left into the purely bosonic configuration by the existence of certain unbroken supersymmetries, that is, the vanishing binding energy of the overlapping brane configuration. To this end, we impose that the mixing terms of the Ricci tensor mentioned above vanish [22]. This happens provided that the following constraints are imposed:

$$B_x C_x^{\bar{q}} X_x^{p_1-2} Y_x^{p_2} W_x^{p_t} = 1\,, \tag{2.11}$$

$$B_y C_y^{\bar{q}} X_y^{p_1} Y_y^{p_2-2} W_y^{p_t} = 1\,. \tag{2.12}$$

These constraints can be physically interpreted as enforcing extremality. They can be read as a way to express W as a function of the other metric components. It is also

interesting to point out now, that all the solutions investigated in Refs. [25, 26, 27] satisfy them, as will be better described below.
After (2.11) and (2.12), we can rewrite Einstein equations as

$$\Box \ln B = \frac{1}{2\sqrt{-g}} \sum_{A=1}^{2} \frac{\xi_A^0}{D-2} S_A (\nabla_\top E_A)^2 \,, \tag{2.13}$$

$$\Box \ln C = \frac{1}{2\sqrt{-g}} \sum_{A=1}^{2} \frac{\xi_A^s}{D-2} S_A (\nabla_\top E_A)^2 \,, \tag{2.14}$$

$$\begin{aligned} &\delta_{ab}\Box \ln X + X^{-2}\left[(p_1-2)\partial_a \ln X \partial_b \ln X + p_2 \partial_a \ln Y \partial_b \ln Y \right.\\ &\left. + \bar{q}\partial_a \ln C \partial_b \ln C + p_t \partial_a \ln W \partial_b \ln W + \partial_a \ln B \partial_b \ln B\right] = \\ &= \tfrac{1}{2\sqrt{-g}}\left[\textstyle\sum_{A=1}^{2} \frac{\xi_A^x}{D-2} S_A (\nabla_\top E_A)^2 \delta_{ab} + S_2 (\partial_a E_2)(\partial_b E_2)\right] \,, \end{aligned} \tag{2.15}$$

$$\begin{aligned} &\delta_{\alpha\beta}\Box \ln Y + Y^{-2}\left[p_1 \partial_\alpha \ln X \partial_\beta \ln X + (p_2-2)\partial_\alpha \ln Y \partial_\beta \ln Y \right.\\ &\left. + \bar{q}\partial_\alpha \ln C \partial_\beta \ln C + p_t \partial_\alpha \ln W \partial_\beta \ln W + \partial_\alpha \ln B \partial_\beta \ln B\right] = \\ &= \tfrac{1}{2\sqrt{-g}}\left[\textstyle\sum_{A=1}^{2} \frac{\xi_A^y}{D-2} S_A (\nabla_\top E_A)^2 \delta_{\alpha\beta} + S_1 (\partial_\alpha E_1)(\partial_\beta E_1)\right] \,, \end{aligned} \tag{2.16}$$

$$\begin{aligned} &(p_1-2)\partial_a \ln X \partial_\beta \ln X + (p_2-2)\partial_a \ln Y \partial_\beta \ln Y + 2\partial_a \ln Y \partial_\beta \ln X \\ &+\bar{q}\partial_a C \partial_\beta C + \partial_a B \partial_\beta B + p_t \partial_a W \partial_\beta W = 0 \,, \end{aligned} \tag{2.17}$$

$$\Box \ln W = \frac{1}{2\sqrt{-g}} \sum_{A=1}^{2} \frac{\xi_A^w}{D-2} S_A (\nabla_\top E_A)^2 \,, \tag{2.18}$$

where we have introduced the symbol $\nabla_\top E_A$ to refer to the gradient of E_A with respect to coordinates relatively transverse to the q_A-brane. It is worth noting that this does not mean at all that functions E_A depend only on those coordinates, as will be clear below. We have also introduced the D'Alembertian, which after (2.11) and (2.12) is simply,

$$\Box = Y^{-2}\nabla_y^2 + X^{-2}\nabla_x^2. \tag{2.19}$$

and the quantities ξ_A^z (z being the label for the different blocks of coordinates), whose value is given by

$$\xi_A^z = \begin{cases} D - q_A - 3 & \text{if } z \text{ is longitudinal to the } q_A\text{-brane}\,, \\ -(q_A+1) & \text{if } z \text{ is transverse to the } q_A\text{-brane}\,. \end{cases} \tag{2.20}$$

Finally, we have defined for convenience the quantities S_A,

$$S_1 = \frac{W^{2p_t} Y^{2(p_2-1)}}{\sqrt{-g}} e^{\epsilon_1 a_1 \phi} \,, \tag{2.21}$$

$$S_2 = \frac{W^{2p_t} X^{2(p_1-1)}}{\sqrt{-g}} e^{\epsilon_2 a_2 \phi} \,, \tag{2.22}$$

where ϵ_A is a positive sign for the electric membranes and a negative sign for the magnetic ones. Note that the metric determinant has a very simple form as a consequence of the 'no-force' conditions (2.11)–(2.12) imposed to the metric,

$$\sqrt{-g} = X_x^2 Y_y^2 \,. \tag{2.23}$$

We must still impose the equations of motion corresponding to the dilaton,

$$\Box\phi = -\frac{1}{2\sqrt{-g}} \sum_{A=1}^{2} \epsilon_A a_A S_A (\nabla_{\top} E_A)^2 \,, \tag{2.24}$$

and the q_A-forms,

$$\partial_\alpha(S_1 \partial_\alpha E_1) = \partial_a(S_1 \partial_a E_1) = 0 \,, \tag{2.25}$$
$$\partial_a(S_2 \partial_a E_2) = \partial_\alpha(S_2 \partial_a E_2) = 0 \,, \tag{2.26}$$

We will finally consider the following ansatz:[4]

$$E_1 = l_1 S_{1x}^{-1} H_1^{-1} \,, \qquad E_2 = l_2 S_{2y}^{-1} H_2^{-1} \,, \qquad S_{1y} = H_1^2 \,, \qquad S_{2x} = H_2^2 \,, \tag{2.27}$$

(with l_1 and l_2 a couple of –up to now– arbitrary constants), that enables us to write everything in terms of a pair of functions H_A which, after eqs. (2.25)–(2.26), must be harmonic,

$$\nabla_y^2 H_1 = \nabla_x^2 H_2 = 0 \,. \tag{2.28}$$

In this way, our solutions are going to be characterized by a pair of harmonic functions corresponding to the same number of constituents branes that participate on the configuration. From the point of view of supergravity, this shall mean that we are dealing with configurations that preserve one quarter of the supersymmetries. The most general solutions to eqs. (2.28) are given by an arbitrary set of superpositions of identical branes *localized* along the relative transverse directions:

$$H_1 = 1 + \sum_j \frac{c_j}{|\vec{y} - \vec{y}_j|^{p_2 - 2}} \,, \tag{2.29}$$
$$H_2 = 1 + \sum_j \frac{d_j}{|\vec{x} - \vec{x}_j|^{p_1 - 2}} \,. \tag{2.30}$$

This is the well-known multicenter solution whose existence is due to the no-force condition [21] that we previously imposed in (2.11)–(2.12). Now, if we demand the set of conditions,

$$W_x^{2p_t} Y_x^{2(p_2-2)} e^{\epsilon_1 a_1 \phi_x} = 1 \,, \tag{2.31}$$
$$W_y^{2p_t} X_y^{2(p_1-2)} e^{\epsilon_2 a_2 \phi_y} = 1 \,, \tag{2.32}$$

[4]We present eqs. (2.27) as an ansatz for simplicity. It is possible to argue that they are forced by the equations of motion following the lines of Ref. [24].

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it is quite easy to solve the dilaton equation of motion,

$$\phi = \sum_{A=1}^{2} \epsilon_A a_A \alpha_A \ln H_A \,, \tag{2.33}$$

as well as the diagonal components of the Einstein equations,

$$\ln B = \ln C = -\sum_{A=1}^{2} \frac{D - q_A - 3}{D-2} \alpha_A \ln H_A \,, \tag{2.34}$$

$$\ln X = -\sum_{A=1}^{2} \frac{\xi_A^x}{D-2} \alpha_A \ln H_A \,, \tag{2.35}$$

$$\ln Y = -\sum_{A=1}^{2} \frac{\xi_A^y}{D-2} \alpha_A \ln H_A \,, \tag{2.36}$$

$$\ln W = \sum_{A=1}^{2} \frac{q_A + 1}{D-2} \alpha_A \ln H_A \,, \tag{2.37}$$

provided $\alpha_A = \frac{1}{2} l_A^2$. The fact that B and C are equal could have been directly predicted from the $\mathrm{SO}(1, \bar{q})$ boost invariance of the extremal configuration. The only equations that remain to be solved are the off-diagonal Einstein equations which at this stage simply reduce to a set of algebraic equations:

$$\left[\frac{(p_2 + p_t - 2)(q_1 + 1)^2 + (q_1 + 1)(D - q_1 - 3)^2}{(D-2)^2} + \frac{1}{2} a_1^2 \right] \alpha_1 = 1 \,, \tag{2.38}$$

$$\left[\frac{(p_1 + p_t - 2)(q_2 + 1)^2 + (q_2 + 1)(D - q_2 - 3)^2}{(D-2)^2} + \frac{1}{2} a_2^2 \right] \alpha_2 = 1 \,, \tag{2.39}$$

and

$$\sum_{A \neq B = 1}^{2} (\bar{q} + 3)(D - q_A - 3)(D - q_B - 3) + p_t (q_A + 1)(q_B + 1)$$
$$-2(p_A - 2)(D - q_A - 3)(q_B + 1) + \frac{1}{2}(D-2)^2 \epsilon_A a_A \epsilon_B a_B = 0 \,. \tag{2.40}$$

From the first two equations, we obtained an explicit value for the α_A's in terms of the dimensions of the constituent branes and the dilaton couplings

$$\alpha_A = \frac{D-2}{\Delta_A} \,, \tag{2.41}$$

where

$$\Delta_A = (q_A + 1)(D - q_A - 3) + \frac{1}{2}(D-2) a_A^2 \,, \tag{2.42}$$

that coincides with the one obtained in the case studied in Ref. [22]. The third equation leads us directly to the announced *rule for localized intersections of p-branes* which is one of the main results of this paper:

$$\bar{q} + 3 = \frac{(q_1 + 1)(q_2 + 1)}{D - 2} - \frac{1}{2} \epsilon_1 \epsilon_2 a_1 a_2 \,. \tag{2.43}$$

This expression is very similar to that of Ref. [22], except for a shift of two units in $\bar{q}$. We will show in the next section, that this rule leads to most known cases of localized intersections of p-branes appearing in the literature (within the scope of our ansatz), and that it does not have further solutions. In that respect, we think that the main interest of eq. (2.43) relies in the fact that it is a unique algebraic expression that collects a complete family of solutions which are related by various dualities and dimensional reductions.

Let us close this section by stressing that the configuration we have obtained so far,

$$B = C = \prod_{A=1}^{2} H_A^{-(D-q_A-3)/\Delta_A}\,, \tag{2.44}$$

$$X = \prod_{A=1}^{2} H_A^{-\xi_A^x/\Delta_A}\,, \tag{2.45}$$

$$Y = \prod_{A=1}^{2} H_A^{-\xi_A^y/\Delta_A}\,, \tag{2.46}$$

$$W = \prod_{A=1}^{2} H_A^{(q_A+1)/\Delta_A}\,, \tag{2.47}$$

$$e^{\phi} = \prod_{A=1}^{2} H_A^{\epsilon_A a_A(D-2)/\Delta_A}\,, \tag{2.48}$$

$$E_1 = \sqrt{2\alpha_1}\, H_1^{-1} H_2^{-2\frac{\xi_2^x-\xi_2^y}{\Delta_2}}\,, \tag{2.49}$$

$$E_2 = \sqrt{2\alpha_2}\, H_1^{2\frac{\xi_1^x-\xi_1^y}{\Delta_1}} H_2^{-1}\,, \tag{2.50}$$

is consistent with the conditions (2.31) and (2.32), and obeys the harmonic superposition rule [19].

3. Localized intersections in various dimensions

In this section, we study the complete set of solutions admitted by eq. (2.43) in various spacetime dimensions. We stress on the duality chains that relate different solutions among themselves. We will use a very convenient notation introduced in Ref. [29], denoting by $(\bar{q}|\mathcal{M}_1, \mathcal{M}_2)$ to the localized overlapping or intersection between an $\mathcal{M}_1$-brane and an $\mathcal{M}_2$-brane with $\bar{q}$ common tangent directions. We will see that the solutions are not new but they correspond to a family of configurations that have been separately considered before by many authors.

3.1. Localized intersections of M-branes

Let us first analyze the eleven dimensional case. There is a 4-form field strength in $D = 11$ supergravity, that can describe either electric M2-branes or magnetic M5-branes. As there is no dilaton, we simply put $a_A = 0$ for the 4-forms. Then, the overlapping rule derived in eq. (2.43) acquires the simpler form:

$$\bar{q} + 3 = \frac{(q_1+1)(q_2+1)}{9}\,, \tag{3.1}$$

whose only solution is $q_1 = q_2 = 5$ and $\bar{q} = 1$, that is, M5-branes overlapping in a string, $(1|M5, M5)$. This is exactly the solution recently obtained in Ref. [25] by rather different means. There, the solution is found by uplifting either overlapping NS5-branes or mutually orthogonal D4-branes to eleven dimensions.

Explicitely, we find $\alpha_A = 1/2$, and a line element given by

$$ds^2 = H_1^{2/3} H_2^{2/3} \left(H_1^{-1} H_2^{-1} (-dt^2 + ds^2) + H_1^{-1} d\vec{x}^2 + H_2^{-1} d\vec{y}^2 + d\omega^2 \right) . \tag{3.2}$$

As explained in Ref. [27], this solution does not satisfy the (p-2)-dimensional self-intersection rule for p-branes [18]. This puzzle is solved by observing that a third brane can be introduced without breaking further supersymmetries [26, 27]. We will discuss this point in the next section, where we will obtain a general rule for the introduction of a third brane inside an overlapping configuration. It would also be interesting to explore what kind of solution appears if localization in the ω-direction is demanded.

3.2. Localized intersections of NS-branes with other branes

The NS-NS sector of string theory is known to posses a 3-form field strength that couples to the dilaton with $a_A = -1$. It can couple to a fundamental string and to a magnetic NS5-brane. The overlapping rule for these objects is:

$$\bar{q} + 3 = \frac{(q_1+1)(q_2+1)}{8} - \frac{1}{2}\epsilon_1\epsilon_2\,. \tag{3.3}$$

It is immediate to see that this equation admits only one solution describing two NS5-branes overlapping in a string, $(1|NS5, NS5)$. This solution can also be obtained from a string-like soliton solution of heterotic string theory first considered in Ref. [28], by setting the gauge fields to zero [25]. Once again $\alpha_A = 1/2$ and the line element is simply:

$$ds^2 = H_1^{3/4} H_2^{3/4} \left(H_1^{-1} H_2^{-1} (-dt^2 + ds^2) + H_1^{-1} d\vec{x}^2 + H_2^{-1} d\vec{y}^2 \right) . \tag{3.4}$$

A common feature of these solutions is the appearance of an overall conformal factor, while each direction tangent to the worldvolume of a q_A-brane gets a factor H_A^{-1} as already noticed in Refs. [26, 27, 29].

Now we look at the localized intersections of NS-branes and D-branes. We use the fact that $a_A = (5 - n_A)/2$ is the coupling to the dilaton of the n_A-form field strengths coming from the RR sector. It is immediate to see that equation (2.43) does not admit localized intersections of a fundamental string and the D-branes. Concerning NS5-branes, the overlapping rule for these objects and Dq-branes is:

$$\bar{q} = q - 3\,, \tag{3.5}$$

which precisely agrees with the case by case result obtained in Ref. [27]. So, the possible overlappings between NS5-branes and D-branes are $(q-3|NS5, Dq)$, for $3 \leq q \leq 8$.

Here we can give two important examples of such overlapping which are going to be better clarified in section 5. The first one is that appearing in the brane setup of Ref. [9] for a configuration which gives $N = 2$ supersymmetry in 3 spacetime dimensions. It consists of a NS5-brane with (12345) spatial directions and a D5-brane with (12789) spatial directions such that the number of common directions are coincident with our previously derived rule (3.5). One would, in principle, have expected that this configuration matches the overlapping rule because, as explained in Ref. [9], it is crucial there that both membranes are well-localized in their relative transverse directions. They are also localized in the overall transverse x_6-direction.

The second example appears in the brane setup of Ref. [10] for a configuration which gives $N = 1$ supersymmetry in 4 spacetime dimensions. In this configuration there are two types of NS5-branes, one denoted by NS5 in the (12345) spatial directions and one denoted by NS5' in the (12389) spatial directions. There are D6-branes in the (123789) spatial directions. The rule (2.43) accounts for the localized intersection between the D6-branes and the NS5-branes, but seems to disagree with the intersection between the D6-branes and the NS5'-brane. Moreover, this last intersection obeys the rule (33) of Ref. [22]. In the next section, we will show that –for the case of localized intersections– we can add new branes with vanishing binding energy whose intersection rule is not given by (2.43). The new intersection rule is precisely that of equation (33) in Ref. [22], and the origin of this coincidence will be clarified.

3.3. Localized intersections of D-branes

The D-branes are the bearers of the RR charges and they give rise to field strengths which couple to the dilaton in such a way that $\epsilon_A a_A = (3 - q_A)/2$ both for electrically and magnetically charged q_A-branes. Then the equation (2.43) gives:

$$2\bar{q} + 8 = q_1 + q_2\,. \tag{3.6}$$

The D-branes shall intersect in such a way that there are eight relative transverse directions, a condition which is also known to be required by the preservation of unbroken supersymmetries [15, 31]. The solution $(0|D4, D4)$ has the following line element,

$$ds^2 = H_1^{5/8} H_2^{5/8} \left(H_1^{-1} H_2^{-1}(-dt^2) + H_1^{-1} d\vec{x}^2 + H_2^{-2} d\vec{y}^2 + d\omega^2 \right)\,. \tag{3.7}$$

It could have also been obtained just by dimensional reduction of $(1|M5, M5)$ on the common string direction. One can obtain the remaining type II solutions by a chain of

T dualities,

$$
\begin{array}{ccc}
(0|D4,D4) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (0|D3,D5) \\
\mathrm{T}\updownarrow & & \mathrm{T}\updownarrow \\
(1|D5,D5) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (1|D4,D6)
\end{array}
. \tag{3.8}
$$

There are more configurations that are, in principle, accesible by this procedure,

$$
\begin{array}{ccccc}
(0|D2,D6) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (0|D1,D7) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (0|D0,D8) \\
\mathrm{T}\updownarrow & & \mathrm{T}\updownarrow & & \mathrm{T}\updownarrow \\
(1|D3,D7) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (1|D2,D8) & \stackrel{\mathrm{T}}{\longleftrightarrow} & (1|D1,D9)
\end{array}
\tag{3.9}
$$

We present them separately for the following reason: the relative transverse space of one of the D-branes, being less or equal than two-dimensional, leads to the appearance of logarithmic or linear singularities. Consider, for example, $(1|D3,D7)$ and see that the harmonic function corresponding to the D7-brane is harmonic in the Euclidean plane. So, each D7-brane produces a conical singularity and its energy per unit 7-volume result to be logarithmically divergent. It is also interesting to mention the case $(1|D2,D8)$, where an harmonic function in one Euclidean coordinate corresponds to the D8-branes thus being piecewise linear.[5] Following a similar reasoning, it is immediate to see that the configuration $(1|D1,D9)$ is equivalent to an isolated D1-brane in empty ten dimensional Minkowski space. In the context of supergravity, all these solutions are known to preserve 1/4 of the original supersymmetries [26, 27]. They are related to the configurations obtained in the previous subsection by S-duality.

4. Adding a third localized p-brane

Let us consider the possibility of adding a third brane into the picture with the following two requirements:

(i) the brane spans the intersecting and totally transverse coordinates, thus being a $(\bar{q}+p_t)$-brane.

(ii) the geometry of the target space gets modified only by the introduction of a third function H_3, that corresponds to the new brane, while the contributions of the overlapping branes are untouched. This is analogous to the 'no-force' condition, in the sense that we shall add a new object that modifies the geometry without exerting any gravitational attraction to the previous configuration. This requirement is related to the existence of a certain amount of unbroken supersymmetries or, in our case, to the elimination of mixing terms in the Ricci tensor.

We define new functions for the line element

$$
\begin{aligned}
ds^2 = & -\hat{B}^2dt^2 + \hat{C}^2\delta_{ij}ds^ids^j + \hat{X}^2\delta_{ab}dx^adx^b + \hat{Y}^2\delta_{\alpha\beta}dy^\alpha dy^\beta \\
& + \hat{W}^2\delta_{\mu\nu}dw^\mu dw^\nu ,
\end{aligned}
\tag{4.1}
$$

[5] The D8-brane is a domain wall solution of a massive version of type IIA supergravity, separating regions of different cosmological constants [32, 33]

such that the contribution of the new brane is given by a new factor $\tilde{\mathcal{F}}(x^a, y^\alpha)$,

$$\hat{\mathcal{F}}(x^a, y^\alpha) \equiv \mathcal{F}(x^a, y^\alpha)\tilde{\mathcal{F}}(x^a, y^\alpha) = \mathcal{F}_x(x^a)\mathcal{F}_y(y^\alpha)\tilde{\mathcal{F}}(x^a, y^\alpha)\,, \tag{4.2}$$

that we allow to depend on the whole set of relatively transverse coordinates. Note the difference with respect to the previous case where we imposed a factorization (2.8) of the coordinates dependence. Here, the coordinates which are relatively transverse to the third brane are x_a's and y_α's.

In order to see how our requirements (i) and (ii) severely constrain the resulting configuration, we compute the Ricci tensor (see Appendix) in the background of the former intersecting brane solution, and impose the 'no-force' conditions –thought of as the vanishing of the mixing terms in the Ricci tensor– as before,

$$\tilde{B}\tilde{C}^{\tilde{q}}\tilde{X}^{p_1-2}\tilde{Y}^{p_2}\tilde{W}^{p_t} = 1\,, \tag{4.3}$$

$$\tilde{B}\tilde{C}^{\tilde{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t} = 1\,. \tag{4.4}$$

From these conditions, it is immediate to see that

$$\tilde{X}^2 = \tilde{Y}^2 \tag{4.5}$$

and

$$\tilde{B}\tilde{C}^{\tilde{q}}\tilde{X}^{p_1+p_2-2}\tilde{W}^{p_t} = 1\,. \tag{4.6}$$

By plugging these conditions back into the Ricci tensor, we can write the equations of motion, in the background of the former overlapping brane configuration. The introduction of the third brane modifies the S_A functions, $S_A \to \hat{S}_A$ with

$$\hat{S}_1 = \tilde{W}^{2p_t}\tilde{X}^{2(p_2-2)}e^{\epsilon_1 a_1\tilde{\phi}}S_1\,, \tag{4.7}$$

$$\hat{S}_2 = \tilde{W}^{2p_t}\tilde{X}^{2(p_1-2)}e^{\epsilon_2 a_2\tilde{\phi}}S_2\,, \tag{4.8}$$

where $\tilde{\phi}$ is the correction to the dilaton also due to the third brane. The only way to solve our new system without changing the values of H_1 and H_2, or the ones of S_1 and S_2, is to impose the following conditions:

$$\tilde{W}^{2p_t}\tilde{X}^{2(p_2-2)}e^{\epsilon_1 a_1\tilde{\phi}} = 1\,, \tag{4.9}$$

$$\tilde{W}^{2p_t}\tilde{X}^{2(p_1-2)}e^{\epsilon_2 a_2\tilde{\phi}} = 1\,, \tag{4.10}$$

which also lead to an expression of the dilaton in terms of the metric function $\tilde{X}$,

$$\tilde{X}^{2(p_1-p_2)} = e^{(\epsilon_1 a_1-\epsilon_2 a_2)\tilde{\phi}}\,. \tag{4.11}$$

Now, the diagonal components of the Einstein equations for a generic function $\mathcal{F} = B, C, W, X, Y$ are simply given by

$$\hat{\Box}\ln\tilde{\mathcal{F}} = \frac{1}{2\sqrt{-\hat{g}}}\frac{\xi_{\mathcal{F}}}{D-2}\hat{S}_3\left[\hat{X}^{-2}(\nabla_x E_3)^2 + \hat{Y}^{-2}(\nabla_y E_3)^2\right]\,, \tag{4.12}$$

where,

$$\hat{S}_3 = \frac{\hat{X}^{2p_1}\hat{Y}^{2p_2}}{\sqrt{-\hat{g}}}\,, \tag{4.13}$$

while the contribution of the third brane to each metric component is given by the factor $\xi_{\mathcal{F}}$,

$$\xi_{\mathcal{F}} = \begin{cases} D-(\bar{q}+p_t)-3 & \text{if } \mathcal{F} \text{ is } B,\, C \text{ or } W\,, \\ -(\bar{q}+p_t+1) & \text{if } \mathcal{F} \text{ is } X \text{ or } Y\,, \end{cases} \tag{4.14}$$

in accordance to the previously established recipe (2.20). In order to solve (4.12), we should set $E_3 = l_3 H_3^{-1}$ and $\hat{S}_3 = H_3^2$. Now, the solution is

$$\ln\tilde{\mathcal{F}} = -\frac{\xi_{\mathcal{F}}}{D-2}\ln H_3^{l_3^2/2}\,, \tag{4.15}$$

if and only if H_3 satisfies

$$(H_2^{-1}\nabla_x^2 + H_1^{-1}\nabla_y^2)H_3 = 0\,, \tag{4.16}$$

thus being non-harmonic.[6] This equation coincides with the one obtained in Refs. [26, 27] for the case of three branes in the context of supergravity. It was also obtained earlier in the context of ten dimensional solutions of string theory representing extreme dyonic black holes [34].

As before, the remaining block-diagonal Einstein equations give the value of $\alpha_3 = l_3^2/2$ whose expression is coincident to that of α_A's previously obtained

$$\alpha_3 = \frac{D-2}{\Delta_3}\,, \tag{4.17}$$

where

$$\Delta_3 = (q_3+1)(D-q_3-3) + \frac{1}{2}(D-2)a_3^2\,.$$

Here, we introduced the dimension of the third brane $q_3 = \bar{q}+p_t$. On the other hand, the off-block-diagonal Einstein equations lead to the *intersection rule for the third brane*,

$$\bar{q}+1 = \frac{(q_i+1)(q_3+1)}{D-2} - \frac{1}{2}a_i a_3 \epsilon_i \epsilon_3\,, \tag{4.18}$$

where $i = 1, 2$ refers to anyone of the 'old' two branes. Again, we should say that the unique algebraic expression to which we arrive is a fingerprint of the various dualities and dimensional reductions that relate the solutions of (4.18) among themselves. We must comment on the fact that the expression for the intersecting dimension is the same as the one obtained in Ref. [22] for intersecting branes that depend in the overall transverse coordinates. The reason of this coincidence will be clarified below.

[6] Note that eq. (4.16) is indeed a curved space Laplace equation. Thus, though H_3 is not harmonic in the flat space sense, it can be thought of to be harmonic in some generalized curved space sense. In fact, this equation appears whenever the effective transverse space is curved [34]. We are grateful to Arkady Tseytlin for his clarifying comments on this issue.

5. Examples with three branes

In this section, we will study the three brane configurations that solve eqs. (2.43) and (4.18). We will introduce a simple notation that generalize that of Ref. [29]: a configuration corresponding to a localized intersection of dimension $\bar{q}$ of an $\mathcal{M}_1$-brane and an $\mathcal{M}_2$-brane, forming a bound state of zero binding energy with a third brane $\mathcal{M}_3$, will be denoted as $(\mathcal{M}_3|\bar{q}|\mathcal{M}_1, \mathcal{M}_2)$. Note that, as discussed above, the fully localized function corresponding to the $\mathcal{M}_3$-brane is, in principle, non-harmonic. We will see that no new solutions emerge from our approach. We will consider certain examples, which are related to well-known brane configurations used to obtain information about dualities and strong coupling effects in supersymmetric gauge theories.

5.1. Three M-branes

We start with the configuration introduced in Refs. [26, 27] and reobtained in subsection 3.1, i.e. with two M5-branes, one in (12345) spatial directions and the other one in (16789) directions. Here, the overall transverse direction is x_{10} and the intersecting direction is x_1. Thus, the third brane should be an M2-brane that spans (1 10) spatial directions. For $q_1 = q_2 = 5, q_3 = 2$ and $a_1 = a_2 = a_3 = 0$ the condition (4.18) is automatically satisfied. The function corresponding to the third brane depends now on both sets of variables which describe the wordvolumes of the two M5-branes, so we have H_3 as a function of $(x^2, \cdots, x^9)$ that satisfies the equation (4.16) which is just the same as the condition (3.8) in [26] or (25) in [27]. By using the notations of formula (3.2) for the groups of coordinates, the line element that corresponds to the $(M2|1|M5, M5)$ configuration is given by:

$$ds^2 = H_1^{2/3} H_2^{2/3} H_3^{1/3} \Big[H_1^{-1} H_2^{-1} H_3^{-1} (-dt^2 + ds^2) + H_1^{-1} d\vec{x}^2 + H_2^{-1} d\vec{y}^2 + H_3^{-1} dw^2 \Big] . \tag{5.1}$$

As explained in Ref. [27], this solution can be thought of as corresponding to an M2-brane being stretched between two M5-branes: when two M5-branes are brought together to intersect on a string, one should think of the intersection as being a collapsed M2-brane. Now we have to observe the following fact: if we start with this configuration and we just take off one of the M5 branes, say the one oriented in (12345) spatial directions, we end with a configuration of an M2-brane (110) and an M5-brane (16789) intersecting on a string. In fact, one can set $H_1 = 1$ in (5.1) to obtain

$$ds^2 = H_2^{2/3} H_3^{1/3} \Big[H_2^{-1} H_3^{-1} (-dt^2 + ds^2) + d\vec{x}^2 + H_2^{-1} d\vec{y}^2 + H_3^{-1} dw^2 \Big] , \tag{5.2}$$

which represents the configuration referred above that, in general, does not satisfy the harmonic superposition principle. In fact,

$$(H_2^{-1} \nabla_x^2 + \nabla_y^2) H_3 = 0 . \tag{5.3}$$

We will use a variant of our notation to call this configuration $(M2|1|M5)$. It is interesting to mention that the intersection string of this configuration is localized in the M5-brane but not in the M2-brane. It has been studied earlier in Refs. [26, 27]. Note that a similar solution with the intersection localized in the M2-brane instead of the M5-brane exists [26] though it cannot be obtained within our approach. Our construction of localized non-harmonic intersections as $(M2|1|M5)$ is asymmetric from the very beginning, a fact reflected on the different nature of H_2 and H_3. We would like to stress on the fact that the amount of supersymmetry is not related to the degree of localization of the membranes participating in a given configuration.

There is a particular solution of this system with H_3 an harmonic function of the x_a's coordinates.[7] Thus, the configuration $M2 \cap M5(1)$ that depends on the overall transverse coordinates [22] is a particular case of the (more general) solution $(M2|1|M5)$. One can 'deform' smoothly $(M2|1|M5)$ into $M2 \cap M5(1)$, an operation that cannot modify a relation between integers as it is the intersection rule previously obtained. It is then clear why we have obtained an intersection rule (4.18) that coincides with the one derived in Ref. [22]. The same reasoning can be straightforwardly applied to the whole set of configurations we are considering in this section.

5.2. Configurations with NS and D branes

As we discussed in subsection 3.2, the only configuration involving localized NS-branes is $(1|NS5, NS5)$. Then, $\bar{q} = 1$, $p_t = 0$ and $q_3 = 1$. It can be easily seen that equation (4.18) is not satisfied for a D1-brane but is precisely an identity for a fundamental string. That is, one can accomodate a fundamental string along the common direction of the solitonic NS-branes with vanishing binding energy. This configuration, whose line element is given by

$$ds^2 = H_1^{3/4} H_2^{3/4} H_3^{1/4} \left[H_1^{-1} H_2^{-1} H_3^{-1}(-dt^2 + ds^2) + H_1^{-1} d\vec{x}^2 + H_2^{-1} d\vec{y}^2 \right] ,$$

should be denoted by $(NS1|1|NS5, NS5)$, and it is clear that it can be uplifted to eleven dimensions yielding (5.1). One can follow the procedure mentioned in the previous subsection to extract one of the NS5-branes ending with the configuration $(NS1|1|NS5)$ first introduced in Ref. [26].

Concerning localized intersection of NS-branes and D-branes, we have shown in subsection 3.2 that the only possible configurations that can be written in terms of harmonic functions are $(q-3|NS5, Dq)$, for $3 \leq q \leq 8$. Then, for these configurations, one has $\bar{q} = q - 3$, $p_t = 1$ and $q_3 = q - 2$. It is immediate to check in (4.18) that a Dq_3-brane can be placed with vanishing binding energy as to build the $(D(q-2)|q-3|NS5, Dq)$ configuration. It is worth to mention that, in spite of the fact that the values $q = 3$ and $q = 7$ give enough room as to introduce a fundamental string and

[7] Also, after this work was completed, an explicit solution of (5.3) was found for a D2-brane (also for a NS5-brane or a wave) localized within a D6-brane, in the region close to the core of the D6-brane [35].

a NS5-brane, they do not satisfy the intersection rule (4.18). Consequently, in our framework, we find that only D-branes can be stretched between a NS5-brane and another D-brane.

At this point, we should come back to the examples that we started to discuss on section 3 which, after the addition of the third localized brane, are very similar to the ones used in [9] for a $N = 2$ configuration and in [10] for a $N = 1$ configuration.

Let us start with the NS5 (12345) - D5 (12789) configuration which preserves 1/4 of the supersymmetry. Then we have the intersection given by (12) and the overall transverse coordinate given by (6). Then we see the possibility of adding a third brane in the (126) direction and this will be a D3-brane. This does not break further supersymmetries. The intersection dimension agrees with formula (4.18). By studying the brane dynamics and the conservation of magnetic charge, the appearance of the D3-brane was explained in [9]. The difference between our case and theirs is that our D3-brane is of infinite extension on x_6 direction whereas their D3-brane is of finite extension on x_6 direction, this extension being just the inverse of the coupling constant of the gauge group $U(N)$ if we have N D3-branes on top of each other. In our case, by having D3-branes with infinite extension on x_6 direction, would give only global groups, with coupling constant zero.

The second example is the one used in [10]. Consider the localized intersection of NS5 (12345) - D6 (123789). The intersection is given by (123) and the overall transverse by (6). Then we see the possibility of introducing a D4-brane in the (1236) direction. Again our D4-brane is of infinite extension in the x_6 direction, as compared with [10] where the D4-branes are finite in that direction. Note, that we are not able to consider within our framework the addition of the NS5'-branes that complete the configuration considered in Ref. [10].

5.3. Configurations with D-branes

It is remarkable that, within our approach, there is a unique type IIB configuration of three D-branes with vanishing binding energy, $(D1|1|D5, D5)$, as can be easily seen from (4.18). There are, of course, other solutions that can be constructed from it by delocalizing in a set of coordinates and applying T-dualities along them [26]. Finally, it is interesting to mention that the intersection rule (4.18) allows to locate a fundamental string in the transverse direction of any of the configurations of section 3 but it does not allow the fundamental string to be in the common direction of a pair of localized D-branes.

6. Conclusions and discussion

In the present paper we discussed configurations with two and three branes which are localized in their relative transverse coordinates. We obtained an intersection rule for the case of two branes by solving the equations of motion in a purely bosonic context. Our intersection rule does not give new solutions and it just collects some of the solutions that have appeared in the literature. It differs from the rule derived in Ref. [22] –corresponding to intersecting branes which are localized in the overall transverse coordinates– by a shift of two units. We considered the introduction of a third brane with vanishing binding energy. We derived the corresponding intersection rule between the first two branes and the third one which happens to be identical to the one of Ref. [22]. We clarified the origin of this coincidence by showing that the configurations which are localized in the overall transverse coordinates can be obtained from those of three branes with localized intersections. All branes are BPS states so for a threshold superposition we used the 'no-force' condition which led to strong simplifications of the equations of motion.

It would be interesting to generalize our work to include non-extreme[8] as well as extreme but non-supersymmetric configurations. In spite of the fact that it is not clear if non-supersymmetric brane configurations represent consistent stable backgrounds, an explicit construction of a generic configuration of this kind could be a first step in order to study non-supersymmetric gauge theories by using brane techniques. As well, it would be very interesting to obtain within our approach other kind of brane configurations which have been recently considered in the literature as p-branes at angles [37] and the so-called (p,q) polymers [38] or (p,q) webs [39] of branes. There are certain critical values for the angles of these configurations which might be thought of as being originated from the 'no-force' conditions obtained from the Ricci tensor.[9] It should also be interesting to understand this kind of intersections as the appearance of a certain soliton in the worldvolume field theory of the complementary p-brane in the way recently introduced in Ref. [29, 40, 41].

Another aspect that deserves a future study is the relation of the kind of configurations appearing in our work and other geometries. The intersecting brane configurations where all the functions depend on the overall transverse coordinates have been related by T-dualities and changes of coordinates with geometries of type $\mathrm{adS}_k \times E^l \times S^m$ [42]. The main observation was that the harmonic functions lose the constant part for a specific choice of transformations and so the geometry is changed from a flat one to an adS_k one. In 11 dimensions, they have started from the general M2 (012) - M5 (023456) solution and have considered the near horizon geometry for which the constant parts of the harmonic functions become negligible. The spacetime factorizes as $\mathrm{ad}S_3 \times E^5 \times S^3$. In our case, we do not have a single radius variable and we could not identify a specific geometry when we neglect the constant term. Also, for the three M-branes configuration $(M2|1|M5, M5)$, the function H_3 is not harmonic so we do not have the case of [42]. We think that it would be very interesting to identify the geometries that are connected with our original ones by T-dualities and changes of coordinates of the near horizon geometry in 11 dimensions. We hope to report on some of these problems elsewhere.

[8] After the completion of this work, a paper appeared covering this subject [36].

[9] We are grateful to Amihay Hanany for his suggestions and comments on this respect.

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A. Ricci tensor of the localized intersections

In this appendix we present the non-vanishing components of the Ricci tensor that corresponds to the overlapping brane metric

$$\begin{aligned} ds^2 = & -B^2dt^2 + C^2\delta_{ij}ds^ids^j + X^2\delta_{ab}dx^adx^b + Y^2\delta_{\alpha\beta}dy^\alpha dy^\beta \\ & + W^2\delta_{\mu\nu}dw^\mu dw^\nu \,, \end{aligned} \tag{A.1}$$

in order to clarify the origin of the 'no-force' condition introduced in this paper. They are:

$$\begin{aligned} R_{00} = & \, Y^{-2}\left[\nabla_y \ln B\cdot\nabla_y \ln(BC^{\bar q}X^{p_1}Y^{p_2-2}W^{p_t}) + \nabla_y^2 \ln B\right] \\ & + X^{-2}\left[\nabla_x \ln B\cdot\nabla_x \ln(BC^{\bar q}X^{p_1-2}Y^{p_2}W^{p_t}) + \nabla_x^2 \ln B\right] \end{aligned} \tag{A.2}$$

$$\begin{aligned} R_{ij} = & \, \delta_{ij}\Big(Y^{-2}\left[\nabla_y \ln C\cdot\nabla_y \ln(BC^{\bar q}X^{p_1}Y^{p_2-2}W^{p_t}) + \nabla_y^2 \ln C\right] \\ & + X^{-2}\left[\nabla_x \ln C\cdot\nabla_x \ln(BC^{\bar q}X^{p_1-2}Y^{p_2}W^{p_t}) + \nabla_x^2 \ln C\right]\Big) \end{aligned} \tag{A.3}$$

$$\begin{aligned} R_{ab} = & \, \delta_{ab}\Big(Y^{-2}\left[\nabla_y \ln X\cdot\nabla_y \ln(BC^{\bar q}X^{p_1}Y^{p_2-2}W^{p_t}) + \nabla_y^2 \ln X\right] \\ & + X^{-2}\left[\nabla_x \ln X\cdot\nabla_x \ln(BC^{\bar q}X^{p_1-2}Y^{p_2}W^{p_t}) + \nabla_x^2 \ln X\right]\Big) \\ & + X^{-2}\Big(\partial_a\partial_b \ln(BC^{\bar q}X^{p_1-2}Y^{p_2}W^{p_t}) + (p_1-2)\partial_a \ln X\partial_b \ln X \\ & - 2\partial_a \ln X\partial_b \ln(BC^{\bar q}X^{p_1-2}Y^{p_2}W^{p_t}) + p_2\partial_a \ln Y\partial_b \ln Y \\ & + \bar q\partial_a \ln C\partial_b \ln C + p_t\partial_a \ln W\partial_b \ln W + \partial_a \ln B\partial_b \ln B\Big) \end{aligned} \tag{A.4}$$

$$\begin{aligned} R_{a\beta} = & -\partial_a \ln Y\partial_\beta \ln BC^{\bar q}W^{p_t}X^{p_1-1} - \partial_a \ln BC^{\bar q}W^{p_t}Y^{p_2-1}\partial_\beta \ln X \\ & + \bar q\partial_a C\partial_\beta C + \partial_a B\partial_\beta B + p_t\partial^a W\partial_\beta W \\ & - \partial_a\partial_\beta \ln X^{p_1-1}Y^{p_2-1}BC^{\bar q}W^{p_t} \end{aligned} \tag{A.5}$$

$$
\begin{aligned}
R_{\alpha\beta} = \delta_{\alpha\beta} \Big(& Y^{-2} \Big[\nabla_y \ln Y \cdot \nabla_y \ln(BC^{\bar q} X^{p_1} Y^{p_2-2} W^{p_t}) + \nabla_y^2 \ln Y \Big] \\
& + X^{-2} \Big[\nabla_x \ln Y \cdot \nabla_x \ln(BC^{\bar q} X^{p_1-2} Y^{p_2} W^{p_t}) + \nabla_x^2 \ln Y \Big] \Big) \\
& + Y^{-2} \Big(\partial_\alpha \partial_\beta \ln(BC^{\bar q} X^{p_1} Y^{p_2-2} W^{p_t}) + p_1 \partial_\alpha \ln X \partial_\beta \ln X \\
& - 2\partial_\alpha \ln Y \partial_\beta \ln(BC^{\bar q} X^{p_1} Y^{p_2-2} W^{p_t}) + (p_2-2)\partial_\alpha \ln Y \partial_\beta \ln Y \\
& + \bar q \partial_\alpha \ln C \partial_\beta \ln C + p_t \partial_\alpha \ln W \partial_\beta \ln W + \partial_\alpha \ln B \partial_\beta \ln B \Big)
\end{aligned} \tag{A.6}
$$

$$
\begin{aligned}
R_{\mu\nu} = \delta_{\mu\nu} \Big(& Y^{-2} \Big[\nabla_y \ln W \cdot \nabla_y \ln(BC^{\bar q} X^{p_1} Y^{p_2-2} W^{p_t}) + \nabla_y^2 \ln W \Big] \\
& + X^{-2} \Big[\nabla_x \ln W \cdot \nabla_x \ln(BC^{\bar q} X^{p_1-2} Y^{p_2} W^{p_t}) + \nabla_x^2 \ln W \Big] \Big) \, .
\end{aligned} \tag{A.7}
$$

If we assume that all functions can be factorized

$$
\mathcal{F}(x^a, y^\alpha) \equiv \mathcal{F}_x(x^a) \mathcal{F}_y(y^\alpha) \, , \tag{A.8}
$$

and impose the ‘no-force’ conditions,

$$
B_x C_x^{\bar q} X_x^{p_1-2} Y_x^{p_2} W_x^{p_t} = 1 \, , \tag{A.9}
$$

$$
B_y C_y^{\bar q} X_y^{p_1} Y_y^{p_2-2} W_y^{p_t} = 1 \, . \tag{A.10}
$$

these components get sensibly simplified as follows:

$$
R_{00} = \Box \ln B \tag{A.11}
$$

$$
R_{ij} = \delta_{ij} \, \Box \ln C \tag{A.12}
$$

$$
\begin{aligned}
R_{ab} = \delta_{ab} \, \Box \ln X + X^{-2} [& (p_1-2)\partial_a \ln X \partial_b \ln X + p_2 \partial_a \ln Y \partial_b \ln Y \\
& + \partial_a \ln B \partial_b \ln B + \bar q \partial_a \ln C \partial_b \ln C + p_t \partial_a \ln W \partial_b \ln W]
\end{aligned} \tag{A.13}
$$

$$
\begin{aligned}
R_{a\beta} = & (p_1-2)\partial_a \ln X \partial_\beta \ln X + (p_2-2)\partial_a \ln Y \partial_\beta \ln Y + \bar q \partial_a C \partial_\beta C \\
& + \partial_a B \partial_\beta B + p_t \partial_a W \partial_\beta W + 2\partial_a \ln Y \partial_\beta \ln X
\end{aligned} \tag{A.14}
$$

$$
\begin{aligned}
R_{\alpha\beta} = \delta_{\alpha\beta} \, \Box \ln Y + Y^{-2} [& p_1 \partial_\alpha \ln X \partial_\beta \ln X + (p_2-2)\partial_\alpha \ln Y \partial_\beta \ln Y \\
+ \; & \partial_\alpha \ln B \partial_\beta \ln B + \bar q \partial_\alpha \ln C \partial_\beta \ln C + p_t \partial_\alpha \ln W \partial_\beta \ln W]
\end{aligned} \tag{A.15}
$$

$$
R_{\mu\nu} = \delta_{\mu\nu} \, \Box \ln W \, . \tag{A.16}
$$

B. Third brane with zero binding energy

Let us now consider an additional contribution to all functions of the metric

$$\begin{aligned} ds^2 =& -\hat{B}^2 dt^2 + \hat{C}^2 \delta_{ij} ds^i ds^j + \hat{X}^2 \delta_{ab} dx^a dx^b + \hat{Y}^2 \delta_{\alpha\beta} dy^\alpha dy^\beta \\ &+ \hat{W}^2 \delta_{\mu\nu} dw^\mu dw^\nu \,, \end{aligned} \tag{B.1}$$

in such a way that

$$\hat{\mathcal{F}}(x^a, y^\alpha) \equiv \mathcal{F}(x^a, y^\alpha) \tilde{\mathcal{F}}(x^a, y^\alpha) \,, \tag{B.2}$$

where $\mathcal{F}$ are the solutions to the Einstein equations corresponding to (A.11)–(A.16). The modified Ricci tensor turns out to be

$$\hat{R}_{MN} = \tilde{X}^{-2} R_{MN} + \tilde{R}_{MN} \,, \tag{B.3}$$

where

$$\begin{aligned} \tilde{R}_{00} =& \hat{Y}^{-2} \left[\nabla_y \ln \hat{B} \cdot \nabla_y \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1} \tilde{Y}^{p_2 - 2} \tilde{W}^{p_t}) + \nabla_y^2 \ln \tilde{B} \right] \\ &+ \hat{X}^{-2} \left[\nabla_x \ln \hat{B} \cdot \nabla_x \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1 - 2} \tilde{Y}^{p_2} \tilde{W}^{p_t}) + \nabla_x^2 \ln \tilde{B} \right] \end{aligned} \tag{B.4}$$

$$\begin{aligned} \tilde{R}_{ij} =& \delta_{ij} \Big(\hat{Y}^{-2} \left[\nabla_y \ln \hat{C} \cdot \nabla_y \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1} \tilde{Y}^{p_2 - 2} \tilde{W}^{p_t}) + \nabla_y^2 \ln \tilde{C} \right] \\ &+ \hat{X}^{-2} \left[\nabla_x \ln \hat{C} \cdot \nabla_x \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1 - 2} \tilde{Y}^{p_2} \tilde{W}^{p_t}) + \nabla_x^2 \ln \tilde{C} \right] \Big) \end{aligned} \tag{B.5}$$

$$\begin{aligned} \tilde{R}_{ab} =& \delta_{ab} \Big(\hat{Y}^{-2} \left[\nabla_y \ln \hat{X} \cdot \nabla_y \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1} \tilde{Y}^{p_2 - 2} \tilde{W}^{p_t}) + \nabla_y^2 \ln \tilde{X} \right] \\ &+ \hat{X}^{-2} \left[\nabla_x \ln \hat{X} \cdot \nabla_x \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1 - 2} \tilde{Y}^{p_2} \tilde{W}^{p_t}) + \nabla_x^2 \ln \tilde{X} \right] \Big) \\ &+ \hat{X}^{-2} \Big(\partial_a \partial_b \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1 - 2} \tilde{Y}^{p_2} \tilde{W}^{p_t}) + (p_1 - 2) \partial_a \ln \hat{X} \partial_b \ln \hat{X} \\ &- 2 \partial_a \ln \hat{X} \partial_b \ln (\tilde{B} \tilde{C}^{\bar{q}} \tilde{X}^{p_1 - 2} \tilde{Y}^{p_2} \tilde{W}^{p_t}) + p_2 \partial_a \ln \hat{Y} \partial_b \ln \hat{Y} \\ &+ \bar{q} \partial_a \ln \hat{C} \partial_b \ln \hat{C} + p_t \partial_a \ln \hat{W} \partial_b \ln \hat{W} + \partial_a \ln \hat{B} \partial_b \ln \hat{B} \\ &- (p_1 - 2) \partial_a \ln X \partial_b \ln X - p_2 \partial_a \ln Y \partial_b \ln Y - \bar{q} \partial_a \ln C \partial_b \ln C \\ &- p_t \partial_a \ln W \partial_b \ln W - \partial_a \ln B \partial_b \ln B) \end{aligned} \tag{B.6}$$

$$\begin{aligned} \tilde{R}_{a\beta} =& -\partial_a \ln \hat{Y} \partial_\beta \ln \hat{B} \hat{C}^{\bar{q}} \hat{W}^{p_t} \hat{X}^{p_1 - 1} - \partial_a \ln \hat{B} \hat{C}^{\bar{q}} \hat{W}^{p_t} \hat{Y}^{p_2 - 1} \partial_\beta \ln \hat{X} \\ &+ \bar{q} \partial_a \ln \hat{C} \partial_\beta \ln \hat{C} + \partial_a \ln \hat{B} \partial_\beta \ln \hat{B} + p_t \partial_a \ln \hat{W} \partial_\beta \ln \hat{W} \\ &- \partial_a \partial_\beta \ln \hat{X}^{p_1 - 1} \hat{Y}^{p_2 - 1} \hat{B} \hat{C}^{\bar{q}} \hat{W}^{p_t} - \bar{q} \partial_a \ln C \partial_\beta \ln C \\ &- \partial_a \ln B \partial_\beta \ln B - p_t \partial_a \ln W \partial_\beta \ln W \end{aligned} \tag{B.7}$$

$$
\begin{aligned}
\tilde{R}_{\alpha\beta} = {}& \delta_{\alpha\beta}\Big(\hat{Y}^{-2}\Big[\nabla_y \ln\hat{Y}\cdot\nabla_y \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t}) + \nabla_y^2 \ln\tilde{Y}\Big] \\
& + \hat{X}^{-2}\Big[\nabla_x \ln\hat{Y}\cdot\nabla_x \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1-2}\tilde{Y}^{p_2}\tilde{W}^{p_t}) + \nabla_x^2 \ln\tilde{Y}\Big]\Big) \\
& + \hat{Y}^{-2}\Big(\partial_\alpha\partial_\beta \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t}) + p_1\partial_\alpha \ln\hat{X}\partial_\beta \ln\hat{X} \\
& - 2\partial_\alpha \ln\hat{Y}\partial_\beta \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t}) + (p_2-2)\partial_\alpha \ln\hat{Y}\partial_\beta \ln\hat{Y} \\
& + \bar{q}\partial_\alpha \ln\hat{C}\partial_\beta \ln\hat{C} + p_t\partial_\alpha \ln\hat{W}\partial_\beta \ln\hat{W} + \partial_\alpha \ln\hat{B}\partial_\beta \ln\hat{B} \\
& - p_1\partial_\alpha \ln X\partial_\beta \ln X - (p_2-2)\partial_\alpha \ln Y\partial_\beta \ln Y - \bar{q}\partial_\alpha \ln C\partial_\beta \ln C \\
& - p_t\partial_\alpha \ln W\partial_\beta \ln W - \partial_\alpha \ln B\partial_\beta \ln B)
\end{aligned} \tag{B.8}
$$

$$
\begin{aligned}
\tilde{R}_{\mu\nu} = {}& \delta_{\mu\nu}\Big(\hat{Y}^{-2}\Big[\nabla_y \ln\hat{W}\cdot\nabla_y \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t}) + \nabla_y^2 \ln\tilde{W}\Big] \\
& + \hat{X}^{-2}\Big[\nabla_x \ln\hat{W}\cdot\nabla_x \ln(\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1-2}\tilde{Y}^{p_2}\tilde{W}^{p_t}) + \nabla_x^2 \ln\tilde{W}\Big]\Big)\,.
\end{aligned} \tag{B.9}
$$

These expressions suggest that the 'no-force' condition that must be satisfied in order that the third brane could be bounded to the old configuration with zero binding energy are:

$$\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1-2}\tilde{Y}^{p_2}\tilde{W}^{p_t} = 1\,, \tag{B.10}$$

$$\tilde{B}\tilde{C}^{\bar{q}}\tilde{X}^{p_1}\tilde{Y}^{p_2-2}\tilde{W}^{p_t} = 1\,. \tag{B.11}$$

The Ricci tensor gets simplified after (B.10) and (B.11) to

$$\tilde{R}_{00} = \hat{\Box}\ln\tilde{B} \tag{B.12}$$

$$\tilde{R}_{ij} = \delta_{ij}\;\hat{\Box}\ln\tilde{C} \tag{B.13}$$

$$
\begin{aligned}
\tilde{R}_{ab} = {}& \delta_{ab}\;\hat{\Box}\ln\tilde{X} + \hat{X}^{-2}\Big((p_1-2)\partial_a \ln\hat{X}\partial_b \ln\hat{X} + p_2\partial_a \ln\hat{Y}\partial_b \ln\hat{Y} \\
& + \bar{q}\partial_a \ln\hat{C}\partial_b \ln\hat{C} + p_t\partial_a \ln\hat{W}\partial_b \ln\hat{W} + \partial_a \ln\hat{B}\partial_b \ln\hat{B} \\
& - (p_1-2)\partial_a \ln X\partial_b \ln X - p_2\partial_a \ln Y\partial_b \ln Y - \bar{q}\partial_a \ln C\partial_b \ln C \\
& - p_t\partial_a \ln W\partial_b \ln W - \partial_a \ln B\partial_b \ln B)
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
R_{a\beta} = {}& (p_1-2)\partial_a \ln\hat{X}\partial_\beta \ln\hat{X} + (p_2-2)\partial_a \ln\hat{Y}\partial_\beta \ln\hat{Y} + \bar{q}\partial_a\hat{C}\partial_\beta\hat{C} \\
& + \partial_a\hat{B}\partial_\beta\hat{B} + p_t\partial_a\hat{W}\partial_\beta\hat{W} + 2\partial_a \ln\hat{Y}\partial_\beta \ln\hat{X} - (\text{unhatted})
\end{aligned} \tag{B.15}
$$

$$
\begin{aligned}
\tilde{R}_{\alpha\beta} = {}& \delta_{\alpha\beta}\;\hat{\Box}\ln\tilde{Y} + \hat{Y}^{-2}\Big(p_1\partial_\alpha \ln\hat{X}\partial_\beta \ln\hat{X} + (p_2-2)\partial_\alpha \ln\hat{Y}\partial_\beta \ln\hat{Y} \\
& + \bar{q}\partial_\alpha \ln\hat{C}\partial_\beta \ln\hat{C} + p_t\partial_\alpha \ln\hat{W}\partial_\beta \ln\hat{W} + \partial_\alpha \ln\hat{B}\partial_\beta \ln\hat{B} \\
& - p_1\partial_\alpha \ln X\partial_\beta \ln X - (p_2-2)\partial_\alpha \ln Y\partial_\beta \ln Y - \bar{q}\partial_\alpha \ln C\partial_\beta \ln C \\
& - p_t\partial_\alpha \ln W\partial_\beta \ln W - \partial_\alpha \ln B\partial_\beta \ln B)
\end{aligned} \tag{B.16}
$$

$$\tilde{R}_{\mu\nu} = \delta_{\mu\nu}\;\hat{\Box}\ln\tilde{W}. \tag{B.17}$$

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